Extended Abstract


1 Introduction

The study of steady-state flow of groundwater is governed by the Laplace equation and has great importance in several problems in civil and environmental engineering related to earth dams, wells, slope stability, etc. Depending on boundary conditions and properties of soil an approximate solution is normally obtained by numerical methods, such as the finite difference method or the popular finite element method, implemented in several commercial computer programs available nowadays.

One of the main aspects that must be considered is the influence on the solution of the tridimensional flow since in most computational studies the problem is represented by a 2D mesh, partly due to the greater availability of bi-dimensional programs, partly due the more complicated process of generating 3D finite element meshes and the required computer resources in terms of RAM memory and CPU times.

In this thesis the main characteristics of groundwater flow are investigated with 2D and 3D numerical models involving temporary dewatering of saturated unconfined and confined aquifers. The engineering cases relate to the construction of a shopping mall in the city of São Paulo (Brazil) and the construction of a small hydroelectric power plant in the state of Mato Grosso (Brazil).

Comparisons of the results are made between numerical analyses and the analytical formulation available for some ideal situations. The numerical solutions were obtained either using the conventional scheme of prescribing a well discharge or an alternative technique which combines infiltration and water extraction in the same well.
2 Numerical model

In the first part of this study two cases of temporary drawdown reported by Jin et al. (2011) and Powrie & Preene (1997) were replicated with the aim to compare the feasibility of our model with respect to results already published in the literature.

A technique called Nozzle-Suction-Infiltration described by Jin et al. (2011) involves the simultaneous pumping and infiltration of water in unconfined aquifers using the same well, with water extraction in the upper portion and infiltration in the bottom of the aquifer. As mentioned by Jin (2011) in the proximity of the injection point the pore pressure rises to create a hydraulic barrier, cutting off the flow between the two portions of the well (extraction and infiltration). In this research a 3D model with dimensions 60m x 60m in the horizontal plane and 20m height was used. A single totally penetrating well in the center of the finite mesh was created and subdivided into two parts, considering water extraction in the upper part and water infiltration through the lower one. The granular soil was a well-graded sand with hydraulic conductivity \( k = 1 \times 10^{-3} \) m/s and porosity \( n = 0.25 \). Figure 1 presents the results obtained from the finite element model considering a parametric variation of the following terms: a) extraction/infiltration rate; b) infiltration depth; c) hydraulic conductivity; d) soil anisotropy (\( k_z \neq k_r \)).
Figure 1 – Extraction/infiltration model considering a parametric variation: a) extraction/infiltration rate; b) infiltration depth; c) hydraulic conductivity; d) soil anisotropy.
The second example deals with a comparative study between analytical solutions and numerical results in a confined aquifer. The rectangular \((a \times b)\) excavation in the middle of the 3D finite element mesh (Figure 2) had its length \(a\) increased at every finite element analysis in order to get different values for the normalized distance to the recharge source \((L_0/a)\). The saturated hydraulic conductivity was \(k = 5 \times 10^{-5}\) m/s and the hydraulic head \((H = 32\text{m})\) was kept constant in all lateral boundaries. The flow rate extracted with the dewatering system installed along the excavation perimeter was also evaluated considering analytical equations for the following situations: a) 2D plane flow; b) 2D radial flow; c) combined plane and radial flow.

![Figure 2 – 3D finite element mesh with a totally penetrating rectangular \((a \times b)\) excavation.](image)

**2.1. Plane flow**

For a rectangular dewatering system with geometric \((a > b)\) and close to a recharge source, the flow could be considered essentially plane with a small part due to the radial flow that occurs near the edges of the excavation. In this case the flow rate can be estimated by the following equation:

\[
Q = 2kD(H - h_w) \left[ \frac{a}{L_0} + \frac{\pi}{\ln \left( \frac{2L_0}{b} \right)} \right]
\]  

(2.1)

where \(D\) refers to the aquifer thickness, \((H-h_w)\) represents the drawdown, \(L_0\) the distance from the recharge source and \(k\) the soil hydraulic conductivity.
If the excavation length $a$ is much greater than its width $b$ ($a>>b$) the flow could be admitted as plane only and equation 2.1 can be rewritten as:

$$Q = 2kD(H - h_w)\left(\frac{a}{L_0}\right)$$ (2.2)

The analyses were carried out for excavations geometries $a/b = 10$, 20, and 50, keeping constant the distance to the recharge source ($L_0 = 50m$) and the prescribed hydraulic drawdown ($H - h_w = 5m$). The flow rate obtained by the finite element model ($Q_{ef}$) was compared with those calculated using the analytical formulation ($Q$). Results are presented in Figure 3.

![Figure 3 - Flow rate variation with normalized recharge distance ($L_0/a$) obtained with the 3D finite element model and analytical equations.](image)

As can be seen in Figure 3, the consideration of an additional flow rate generated from the edges of the excavation tends to overestimate the flow with respect to the finite element results, mainly for values of the recharge source distance $L_0/a > 0,1$. When applying the hypothesis of plane flow only (Eq. 2.2) the analytical formulation overestimates the flow rate moderately, ranging from approximately 20% for $L_0/a = 0,01$ to 50% for $L_0/a = 0,1$ and then remaining constant up to $L_0/a = 1$. It may be concluded that as far the recharge source is located, the greater the influence of the excavation geometry ($a/b$) on the plane flow conditions.
2.2. Radial flow

For an excavation with similar dimensions \((a \approx b)\) and a recharge source relatively distant, the flow turns essentially radial and the problem could be analyzed as a circular well with an equivalent radius. In this case the flow rate can be calculated by Eq. 2.3 (Thiem, 1906).

\[
Q = \frac{2\pi k D (H-h_w)}{\ln \left( \frac{L_o}{r_e} \right)}
\]  

(2.3)

In this analysis the drawdown value \((H - h_w = 5 \text{m})\) was kept constant, considering three excavation geometries \((a/b = 1, 2, 5)\) while varying the distance to the recharge source \(a \leq L_o \leq 20a\). The equivalent well radius \((r_e)\) was determined in two ways: assuming an equivalent circular excavation area or an equivalent circular excavation perimeter. Figure 4 presents a comparison of the flow rates calculated with analytical formulation \((Q)\) and the finite element method \((Q_{ef})\). The numerical results were determined by 2D (Powrie e Preene, 1997) and 3D (this research) finite element analyses.

It was observed that the analytical solution approaches the numerical results (2D and 3D cases) for values of recharge source distances between \(7a < L_o < 10a\). The flow rate is underestimated for \(L_o > 10a\) and overestimated for \(L_o < 7a\). The way the equivalent well radius was determined shows irrelevant effects on the results and a minor influence of excavation geometry \(a/b\) was detected for values \(L_o < 5a\), i.e. only when the recharge contour \(L_o\) is close enough of the excavation.
2.2. Radial and plane flow

For a rectangular excavation next to the recharge source, the flow is plane with a radial flow contribution at the corners. In this situation, Cedergren (1989) suggested the following modifications in equations Eqs. 2.1 and 2.2,

\[ Q = 2kD(H - h_w) \left[ \frac{a+b}{L_0} + \pi \right] \]  
(2.4)

\[ Q = 2kD(H - h_w) \left[ \frac{a+b}{L_0} \right] \]  
(2.5)

Figure 5 presents the results (analytical flow rate \( Q \), calculated with equations 2.4 and 2.5, normalized with respect to the numerical flow rate \( Q_{ef} \)) for rectangular excavations of geometries \( a/b = 1, 2, 5 \) situated from a recharge source at distances between \( 0,1a < L_0 < 10a \). The following conclusions can be drawn:

a) For \( L_0/a < 0,2 \) there are significant differences between analytical and numerical results, since the flow occurs in the condition of plane flow essentially;

b) As the distance \( L_0/a \) becomes larger, the flow calculated from equation 2.5 results smaller, indicating that the flow is turning gradually to radial;
c) The excavation geometry \((a/b)\) has a greater influence on results obtained with the plane flow hypothesis than with the combined plane and radial assumption.

d) The application of either Eqs. 2.4 or 2.5 overestimates the flow rate \(Q\) when \(L_0/a < 1\);

e) When \(L_0/a > 1\) the radial and plane flow hypothesis gives errors greater than 150%.

![Figure 5 – Comparative flow rates calculated by analytical equations and 3D finite elements analyses.](image)

3 Engineering cases

3.1. Brooklin Shopping Mall

The Brooklin Shopping Mall is located in the California Street, in the city of Sao Paulo (Brazil). It was built in an area of 736 m\(^2\) equivalent to a rectangle of 46m length and 16m width. The field tests indicated the existence of two soil layers: the upper one composed by soft clayey sand, from the natural ground level to 3m deep, followed by a grey silty sand up to the depth of 19m (borehole limits). The phreatic level was found 3 m from the natural ground level. Building construction requirements specified drawdown of the phreatic level to a depth of 6,5m or, in other words, required an effective drawdown of 3,5m from the original water level. The pumping system was based on 62 wellpoints, spaced 2m each, with a total flow rate \(Q = 0,0156\) m\(^3\)/s measured in the field.
3.1.1.
Analytical solution

The first analysis was made in terms of analytical formulation, using Eq. 3.1, and considered a circular well with equivalent excavation area. The recharge source was assumed as radial, using Eq. 3.2 to determine its distance from the center of the well.

\[
Q = \frac{\pi k (H^2 - h_w^2)}{\ln\left(\frac{R_o}{r_e}\right)}
\]

\[
R_o = 3000(H - h_w)\sqrt{k}
\]

where \(H = 16\) m is the initial hydraulic head in the unconfined aquifer, \(k = 3 \times 10^{-5}\) m/s the isotropic hydraulic conductivity and \(s = H - h_w = 1,2\) m the drawdown calculated with Eq. 3.1.

Another analytical possibility when considering a rectangular excavation \((a/b > 1.5)\) in unconfined aquifers is to add the radial flow near the edges to the plane flow in the middle region, as given by Eq. 3.3.

\[
Q = \frac{\pi k (H^2 - h_w^2)}{\ln\left(\frac{R_o}{r_e}\right)} + 2 \left[ \frac{xk (H^2 - h_w^2)}{2L_o} \right]
\]

where \(x\) is the excavation length \((x = 46\) m) and \(R_o\) determined from Eq. 3.2. As mentioned by Powers (2007) a linear recharge source will produce the same recharge effect such as a radial source situated at twice the distance, i.e. \(R_o = 2L_o\). With this considerations the calculated drawdown was \(s = 1,5\) m.

3.1.2.
2D numerical solution

The problem was analyzed using the finite element program Plaxis 2D v.2013 under the assumption of plane and axisymmetric flow. For the plane flow model, the first soil layer of clayey sand was not considered because the groundwater level passes through its base. The saturated hydraulic conductivity of the aquifer is \(k = 3 \times 10^{-5}\) m/s (Velloso, 1977; Huertas 2006) and the analysis
considered two orthogonal cross sections as shown in Figure 6. The computed drawdown from the finite element analysis was $s = 3\text{m}$.

Figure 6 – 2D finite element analyses: a) cross section along the excavation length; b) cross section along the excavation width; c) drawdown across the middle of the excavation length; d) drawdown across the middle of the excavation width.
The axisymmetric model and its results are presented in Figure 7, with final drawdown \( s = 2 \text{m} \).

![Diagram of axisymmetric flow model](image)

**Figure 7 – Axisymmetric flow model:** a) finite element mesh and boundary conditions; b) axisymmetric drawdown.

### 3.1.3. 3D numerical solution

The 3D model is presented in Figure 8, with 62 wellpoints installed around the rectangular excavation area (46m x 16m). The individual pumping flow rate was \( Q = 0.0156/62 = 2.516 \times 10^{-4} \text{ m}^3/\text{s} \) and a constant hydraulic head \( (H = 16 \text{m}) \) was imposed on the lateral boundaries of the mesh while the bottom surface was considered impervious to flow. The computed drawdown was \( s = 2.3 \text{m} \) (Figure 8b) not enough for the construction purposes \( (s = 3.5 \text{m}) \).

Table 1 presents a summary of the drawdown values for the different analyses.
Table 1 – Summary of the drawdown values

<table>
<thead>
<tr>
<th>Type of analysis</th>
<th>Flow rate (m$^3$/s)</th>
<th>Drawdown (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Field measurements</td>
<td>0.0156</td>
<td>3.5</td>
</tr>
<tr>
<td>Equivalent well radius</td>
<td>0.0156</td>
<td>1.2</td>
</tr>
<tr>
<td>Equivalent well radius including radial flow near the excavation edges</td>
<td>0.0156</td>
<td>1.5</td>
</tr>
<tr>
<td>2D finite element model – plane flow</td>
<td>0.0156</td>
<td>3.0</td>
</tr>
<tr>
<td>2D finite element model – axisymmetric flow</td>
<td>0.0156</td>
<td>2.0</td>
</tr>
<tr>
<td>3D finite element model</td>
<td>0.0156</td>
<td>2.3</td>
</tr>
</tbody>
</table>

Figure 8 – 3D finite element analysis: a) mesh and boundary conditions; b) drawdown in the middle of excavation.

3.1.4. Comments

The numerical plane flow model yielded a drawdown close to the measured value in field, but this kind of analysis does not include the possibility of flow perpendicular to the considered cross section. Similarly, an axisymmetric model
cannot provide accurate results since the excavation dimensions \((a, b)\) are quite different from each other to be accurately represented as a circular well.

However, the 3D finite element model provided a lower drawdown than the actual field measurements, which could be possibly attributed to an inadequate value of the hydraulic conductivity for the saturated aquifer.

### 3.2. Small hydroelectric plant Garganta da Jararaca

The small hydroelectric plant Garganta da Jararaca is located in the Sangue River in the state of Mato Grosso (Brazil). For the construction of the power house an area of 5072 m\(^2\) was excavated, equivalent to a rectangle (60m x 85m) with its smallest side only 6m distant from the river. The excavation reached the depth of 36m below the natural ground level.

From geotechnical tests (SPT and water infiltration tests) the soil profile was characterized by a 6m thick residual soil layer followed by fine sandstone between elevations 390m to 347m (borehole limit). The aquifer (second soil layer) is unconfined with phreatic surface in El. 385m and average hydraulic conductivity \(k = 2.4 \times 10^{-5}\) m/s. Between El. 368m – 365m it was found a very weathered sandstone layer.

For the power house construction it was necessary to lower the groundwater to El. 360m in the middle of the excavation, approximately 25m below the original water level. For this purpose a pumping system composed of 84 deep penetrating wells (up to El. 345m) was installed along the perimeter of the excavation, as shown in Figure 9. Since the field tests did not detect an impervious base within the borehole limits, it was assumed in the next analytical and numerical calculations that the aquifer extended to El. 275m (110m thick).

#### 3.2.1. Analytical solution

The analysis was developed with the concept of equivalent radius well considering the Sangue River (El. 378m) as a linear recharge source at only 6m distance from the excavation border. Using the following equation the drawdown was calculated in the middle of the excavation:
\[ Q = \frac{\pi k (H^2 - h_m^2)}{\ln \left( \frac{2L_0}{r_e} \right)} \quad (3.4) \]

where \( Q \) is the maximum flow rate measured in field \((Q = 1399 \text{ m}^3/\text{h})\) and \( k = 2.4 \times 10^{-5} \text{ m/s} \) is hydraulic conductivity. The drawdown was determined as \( s = 23.6 \text{m} \), a value close to the field measurement \((s = 25 \text{m})\).

### 3.2.2. 2D numerical solution – axisymmetric flow

In the axisymmetric finite element model the Sangue river was assumed as the recharge source for the homogeneous and isotropic aquifer with hydraulic conductivity \( k = 2.4 \times 10^{-5} \text{ m/s} \). The flow rate distributed along the excavation perimeter was considered as \( q = 0.39/2\pi = 0.062 \text{ m}^3/\text{s/\,rad} \). Figure 10 presents the model mesh, boundary conditions and drawdown results with \( s = 25 \text{m} \) in the middle of excavation.

![Figure 9](image1.png)  
**Figure 9** – The drawdown system at Garganta da Jararaca power plant.

![Figure 10](image2.png)  
**Figure 10** – Axisymmetric model for the Garganta da Jararaca power plant (left) and drawdown results (right).
3.2.3. 3D numerical solution

The finite element mesh may be seen in Figure 11a, with all the 84 wells represented to the same depth reached in the field (36m from the natural ground level) and spaced at every 3m. The total flow rate \( Q = 0.39 \ m^3/s \) was divided equally for each well \( q = 16.65 \ m^3/h \). Field tests indicated that the water level has a moderate inclination toward the Sangue river and for this reason the hydraulic heads prescribed on the boundaries are different to each other: at El. 378m on the boundary correspondent to the river bank and at El. 385m for the opposite boundary situated 410m away (Figure 11b). The maximum drawdown computed with the 3D finite element model was \( s = 16m \) (Figure 12).

Table 2 presents all drawdown values determined with the analytical formulation and finite element analyses (2D and 3D models).

![Figure 11 - 3D model of Garganta da Jararaca power plant: a) finite element mesh; b) groundwater configuration before drawdown.](image)
Figure 12 – Groundwater after dewatering: a) horizontal cross section close to the excavation bottom; b) vertical cross section in the middle of excavation.

Table 2 – Drawdown values in the middle of the excavation for Garganta da Jararaca power plant.

<table>
<thead>
<tr>
<th>Type of analysis</th>
<th>Drawdown (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Field measurements</td>
<td>27</td>
</tr>
<tr>
<td>Finite Elements Method 3D</td>
<td>16</td>
</tr>
<tr>
<td>Finite Elements Method 2D (axisymmetric analysis)</td>
<td>25</td>
</tr>
<tr>
<td>Analytical formulation</td>
<td>23.6</td>
</tr>
</tbody>
</table>

3.2.4. Comments

The numerical analysis in the axisymmetric condition reached the best approximation with respect to field measurements, in agreement with previous results obtained by Corrêa (2006).

However, the results from numerical 3D analysis were quite different and this discrepancy could be attributed to several uncertainties such as the pumping flow rate at each well and the hydraulic conductivity of the aquifer. In comparison with the 2D model, the flow incoming perpendicular to the plane of analysis generates higher values of flow rate and, consequently, lower values of drawdown. The purpose of the 3D simulation was also to verify the influence of this component of flow in addition to the effects of the excavation geometry (\(a \neq b\)), the different distances to the recharge boundaries and the slope of the original groundwater level.

Keywords

Drawdown; 3D modeling; aquifer; finite elements.